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## Forced vibrations of laterally loaded piles

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### Abstract

An analytical approach for the determination of the response of a single circular cylindrical pile subjected to a lateral dynamic load is presented. The kinetic and the potential energies of the pile-foundation system are minimized by variational principle to obtain the governing field equations of the pile-foundation system along with the appropriate boundary conditions. A non-dimensional parameter  $\gamma$ , associated with the characteristics of the pile, the foundation and the loading is used to represent the elastic medium. This parameter  $\gamma$  can be determined by using an iterative procedure. The classical finite difference method is used to solve the governing field equations of the pile-foundation system. The validation of the proposed model is demonstrated by applying to several published field pile load tests. Parametric studies with regard to the frequency response of the pile head and the resonant frequency of the pile-foundation system are presented.

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### 1. Introduction

Lateral vibration of piles is an important consideration in the design of piled structures subject to dynamic excitations due to earthquake, wind, operation of machines and waves in offshore environments. In the past two decades, various models have been used to take the soil-pile interaction in the dynamic response analysis of pile foundations. Among those, Winkler models are the simplest and numerically most efficient ones. In Winkler models, the soil is viewed as distributed springs and dashpots that are constant or frequency dependent or as lumped springs concentrated at a finite number of nodes. The spring constants are obtained from analytical considerations or from experimental data. The major advantages of this approach lies in its ability to simulate nonlinearity, inhomogeneity, and hysteretic degradation of the soil surrounding the pile by simply changing the spring and dashpot constants. The Matlock (or Penzien) model (Matlock et al., 1978) and the Novak model (Novak, 1974) may be classified as the conventional Winkler models often used in the dynamic response analysis of pile foundations, while the Nogami

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model (Nogami et al., 1988; 1991) is a Winkler model recently developed for the dynamic and nonlinear response analysis.

Finite element analyses of piles have been carried out by Kuhlemeyer (1979) and Krishnan et al. (1983) in which the piles are represented by axisymmetric elements and energy-absorbing boundaries are used to represent the far field. Considering the obvious limitations of the finite element method for modeling boundaries at infinity, these analyses represent considerable achievements in characterizing the dynamic behavior of piles.

Boundary element formulation has been used by Kaynia and Kausel (1982), Sen et al. (1985), and Banerjee et al. (1987) for the dynamic analysis of piles. Boundary element method offer advantages over other methods primarily because of its ability to take into account the three-dimensional effects of soil continuity and boundaries at infinity. But the major problem is the accuracy of the numerically constructed dynamic solutions since the convergence of the semi-infinite integral is dependent on the frequency parameter.

In this paper, based on the analysis of elastic continuum, a two-parameter model for dynamic analysis of laterally loaded piles is presented. A numerical approach based on the Vlasov method is introduced for the dynamic analysis of soil-pile system. The Vlasov model has been used for the static and dynamic analyses of plates on elastic foundations (Vlasov and Leontiev, 1966; Jones and Xenophontos, 1977; Jones and Mazumdar, 1980; Scott, 1981; Sargand et al., 1987). Vallabhan and Das (1988; 1991a;b) proposed a modified Vlasov model for the static analysis of beams on elastic foundations. Sun (1994) used the Vlasov model for the static analysis of laterally loaded piles. The basic ideas from Sun and Vallabhan and Das, are adopted in this paper. The proposed method uses Hamilton's principle to derive the governing equations of the pile and soil system. A parameter ( $\gamma$ ) associated with the characteristics of the pile, the soil and the loading is used in this model, and can be determined by using an iterative procedure. Finite difference method (FDM) is used for solving the differential equations. The proposed model is verified against two static load test piles and then applied to one dynamic test pile.

## 2. Basic equations

The inherent assumptions in the proposed model are:

- (1) The pile is vertical, elastic, and circular in cross section.
- (2) The pile is perfectly connected to the soil. There is no slippage or separation at the interface of the pile and the surrounding soil.
- (3) The soil is a semi-infinite, elastic, homogeneous, isotropic medium.

A typical circular pile of length  $L$ , radius  $R$  and flexibility  $E_p I_p$  is shown in Fig. 1. The surrounded soil has Young's modulus  $E_g$  and Poisson's ratio  $\nu_g$ . For the soil, vertical displacement associated with laterally loaded pile  $w_g$ , is considered negligible and the horizontal displacements,  $u_g$  and  $v_g$  are approximated by separable functions of the cylindrical coordinate  $r$ ,  $\theta$  and  $z$  (Sun, 1994). That is:

$$u_g(r, \theta, z, t) = u(z, t)\phi(r) \cos \theta \quad (1a)$$

$$v_g(r, \theta, z, t) = -u(z, t)\phi(r) \sin \theta \quad (1b)$$

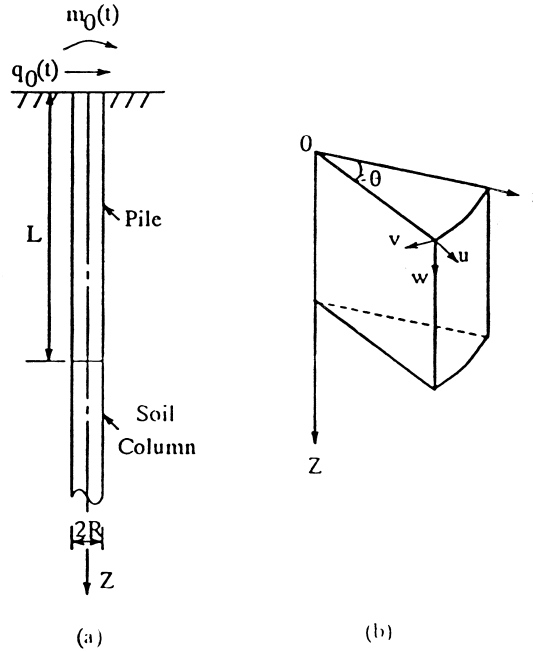


Fig. 1. (a) Pile-soil system; (b) coordinate system and displacement components.

$$w_g(r, \theta, z, t) = 0 \tag{1c}$$

where  $u(z, t)$  = lateral displacement of the pile; and  $\phi(r)$  = dimensionless function representing the variation of the soil displacement in the  $r$ -direction.

The potential energy of the pile and soil system is

$$U = \frac{1}{2} \int_R^\infty \int_0^{2\pi} \int_0^\infty (\sigma_{rr}\epsilon_{rr} + \sigma_{\theta\theta}\epsilon_{\theta\theta} + \tau_{r\theta}\gamma_{r\theta} + \tau_{rz}\gamma_{rz} + \tau_{\theta z}\gamma_{\theta z}) r \, dr \, d\theta \, dz + \frac{1}{2} \int_0^L E_p I_p \left(\frac{d^2 u}{dz^2}\right)^2 dz + \frac{1}{2} \int_L^\infty \pi R^2 G_s \left(\frac{du}{dz}\right)^2 dz \tag{2a}$$

Based on the assumption of displacements given in eqn (1) and using stress-strain relations, eqn (2a) can be rewritten as:

$$U = \frac{1}{2} \int_R^\infty \int_0^{2\pi} \int_0^\infty \left[ \pi(\lambda_s + 3G_s) u^2 \left(\frac{d\Phi}{dr}\right)^2 + 2\pi G_s \left(\frac{du}{dz}\right)^2 \Phi^2 \right] r \, dr \, dz + \frac{1}{2} \int_0^L E_p I_p \left(\frac{d^2 u}{dz^2}\right)^2 dz + \frac{1}{2} \int_L^\infty \pi R^2 G_s \left(\frac{du}{dz}\right)^2 dz \tag{2b}$$

The kinetic energy of the pile and soil system is

$$\begin{aligned}
 T &= \frac{1}{2} \int_0^L m_p \left( \frac{du}{dt} \right)^2 dz + \frac{1}{2} \int_L^\infty m_s \left( \frac{du}{dt} \right)^2 dz + \frac{1}{2} \int_0^{2\pi} \int_0^\infty \int_R^\infty \rho_s \left[ \left( \frac{du_s}{dt} \right)^2 + \left( \frac{dv_s}{dt} \right)^2 \right] r dr d\theta dz \\
 &= \frac{1}{2} \int_0^L m_p \left( \frac{du}{dt} \right)^2 dz + \frac{1}{2} \int_L^\infty m_s \left( \frac{du}{dt} \right)^2 dz + \pi \int_0^\infty \int_R^\infty \rho_s \left( \frac{du}{dt} \right)^2 \theta^2 r dr dz
 \end{aligned} \quad (3)$$

Work of non-conservative force is

$$W_{nc} = q_0(t)u(o, t) + m_0(t) \frac{du(o, t)}{dz} \quad (4)$$

Here  $q_0(t)$  and  $m_0(t)$  are the lateral force and bending moment, respectively, acting on the pile at the ground level  $z = 0$ .

Considering Hamilton's principle

$$\int_{t_1}^{t_2} (\delta T - \delta U) dt + \int_{t_1}^{t_2} \delta W_{nc} dt = 0 \quad (5)$$

using the following notations:

$$m_1 = \int_R^\infty r \Phi^2 dr \quad (6a)$$

$$m_2 = \int_R^\infty r \left( \frac{d\Phi}{dr} \right)^2 dr \quad (6b)$$

$$n_1 = \int_0^\infty \left( \frac{du}{dt} \right)^2 dz \quad (6c)$$

$$n_2 = \int_0^\infty \left( \frac{du}{dz} \right)^2 dz \quad (6d)$$

and

$$n_3 = \int_0^\infty u^2 dz \quad (6e)$$

Collecting the coefficients of  $\delta u$  and  $\delta(du/dz)$  for  $0 \leq z \leq L$ , we get the following eqns and boundary conditions:

$$E_p I_p \frac{d^4 u}{dz^4} - 2c \frac{d^2 u}{dz^2} + ku + m \frac{d^2 u}{dt^2} = 0 \quad (0 \leq z \leq L) \quad (7)$$

where

$$2c = 2\pi G_s m_1 \quad (8a)$$

$$k = \pi(\lambda_s + 3G_s)m_2 \quad (8b)$$

$$m = m_p + 2\pi\rho_s m_1 \tag{8c}$$

with boundary conditions:

$$\left( E_p I_p \frac{d^3 u}{dz^3} - 2c \frac{du}{dz} - q_0 \right) \delta u = 0 \quad (z = 0) \tag{9a}$$

$$\left( E_p I_p \frac{d^2 u}{dz^2} + m_0 \right) \frac{du}{dz} = 0 \quad (z = 0) \tag{9b}$$

$$\left[ \left( E_p I_p \frac{d^3 u}{dz^3} - 2c \frac{du}{dz} \right) \Big|_{\text{Pile}} - c_s \frac{du}{dz} \Big|_{\text{Soil column}} \right] \delta u = 0 \quad (z = L) \tag{9c}$$

$$E_p I_p \frac{d^2 u}{dz^2} \cdot \delta \left( \frac{du}{dz} \right) = 0 \quad (z = L) \tag{9d}$$

For  $L \leq z < \infty$ , we have

$$c_s \frac{d^2 u}{dz^2} - ku - \bar{m}_s \frac{d^2 u}{dt^2} = 0 \quad (L \leq z < \infty) \tag{10}$$

where

$$c_s = \pi R^2 G_s + 2\pi G_s m_1 \tag{11a}$$

$$\bar{m}_s = m_s + 2\pi\rho_s m_1 \tag{11b}$$

Boundary conditions at  $z = L$  and  $z \rightarrow \infty$ :

$$c_s \frac{du}{dz} \delta u = 0 \tag{12}$$

Collecting the coefficients of  $\delta\phi$  for  $R \leq r \leq \infty$ , we have

$$r \frac{d^2 \Phi}{dr^2} + \frac{d\Phi}{dr} - \left( \frac{\gamma}{R} \right)^2 r \Phi = 0 \tag{13}$$

where

$$\left( \frac{\gamma}{R} \right)^2 = \frac{2(G_s n_2 - \rho_s n_1)}{(\lambda_s + 3G_s) n_3} = \frac{2 \left[ G_s \int_0^\infty \left( \frac{du}{dz} \right)^2 dz - \rho_s \int_0^\infty \left( \frac{du}{dt} \right)^2 dz \right]}{(\lambda_s + 3G_s) \int_0^\infty u^2 dz} \tag{14}$$

The solution to eqn (13) is

$$\Phi(r) = \frac{k_0(\gamma r/R)}{K_0(\gamma)} \tag{15}$$

which satisfies the boundary conditions at  $r = R$  and  $r \rightarrow \infty$ :  $\phi(R) = 1$  and  $\phi(\infty) = 0$ .  $K_0()$  denotes the modified Bessel function of the second kind of order zero.

### 3. Equations and boundary conditions for steady-state harmonic loading

It is assumed that the pile is undergoing a steady-state harmonic motion. So let

$$q_0(t) = Q_0 e^{i\Omega t} \quad (16a)$$

$$m_0(t) = M_0 e^{i\Omega t} \quad (16b)$$

$$u(z, t) = u_{pp}(z) e^{i\Omega t} \quad (0 \leq z \leq L) \quad (16c)$$

$$u(z, t) = u_{ps}(z) e^{i\Omega t} \quad (L \leq z < \infty) \quad (16d)$$

where  $\Omega$  = the circular frequency;  $Q_0$  = the amplitude of the lateral load  $q(t)$ ;  $M_0$  = the amplitude of the moment  $m_0(t)$ ;  $u_{pp}(z)$  = the amplitude of pile displacement; and  $u_{ps}(z)$  = the amplitude of the soil column displacement below the pile toe.

Substitution of eqn (16) into eqn (7) gives

$$E_p I_p \frac{d^4 u_{pp}}{dz^4} - 2c \frac{d^2 u_{pp}}{dz^2} + (k - m\Omega^2) u_{pp} = 0 \quad (17)$$

with boundary conditions at  $z = 0$ .

$$E_p I_p \frac{d^3 u_{pp}}{dz^3} - 2c \frac{du_{pp}}{dz} = Q_0 \quad (18a)$$

$$E_p I_p \frac{d^2 u_{pp}}{dz^2} + M_0 = 0 \quad (\text{free-head}) \quad (18b)$$

$$\frac{du_{pp}}{dz} = 0 \quad (\text{fixed head}) \quad (18c)$$

For clamped pile, boundary conditions at  $z = L$  are:

$$u_{pp} = 0 \quad (\text{clamped pile}) \quad (19a)$$

$$\frac{du_{pp}}{dz} = 0 \quad (\text{clamped pile}) \quad (19b)$$

For floating pile, to get the boundary conditions at  $z = L$ , we have to solve eqn (10) at first. Substitution of eqn (16) into eqn (10) gives

$$\frac{d^2 u_{ps}}{dz^2} - \alpha^2 u_{ps} = 0 \quad (20)$$

where

$$\alpha^2 = \frac{k - \bar{m}_s \Omega^2}{c_s} \tag{21}$$

Solving the differential equation given in eqn (17), with the boundary conditions  $u_{ps}(\infty) = 0$  and  $u_{ps}(L) = u_{pp}(L)$ , we get the following solution

$$u_{ps} = u_{pp}(L)e^{-\alpha(z-L)} \tag{22}$$

So the boundary conditions for floating pile at  $z = L$  are

$$\frac{d^2 u_{pp}}{dz^2} = 0 \quad (\text{floating pile}) \tag{19c}$$

$$E_p I_p \frac{d^3 u_{pp}}{dz^3} - 2c \frac{du_{pp}}{dz} - \sqrt{c_s(k - \bar{m}_s \Omega^2)} u_{pp} = 0 \quad (\text{floating pile}). \tag{19d}$$

The parameters  $\gamma$  can be expressed in terms of  $u_{pp}$  as follows:

$$\left(\frac{\gamma}{R}\right)^2 = \frac{2 \left[ G_s \int_0^L \left(\frac{du_{pp}}{dz}\right)^2 dz + \rho_s \Omega^2 \int_0^L u_{pp}^2 dz \right] + N}{(\lambda_s + 3G_s) \int_0^L u_{pp}^2 dz + D} \tag{23}$$

where

$$N = \left( G_s \alpha + \frac{\rho_s \Omega^2}{\alpha} \right) u_{pp}^2(L) \tag{24a}$$

$$D = \left( \frac{\lambda_s + 3G_s}{2\alpha} \right) u_{pp}^2(L) \tag{24b}$$

After a careful examination of the equations and parameters, it can be seen that the solution of a pile in the elastic media is controlled by the non-dimensional parameter  $\gamma$ . This parameter depends on the characteristics of the pile, the soil and the loading. The determination of this parameter is discussed in the following.

#### 4. Proposed methodology

A solution to the problem is sought by satisfying the governing differential equation (eqn 17) subject to the boundary conditions specified by eqns (18) and (19). To obtain the solution for  $u_{pp}$  the value of the parameter  $\gamma$  defined by eqn (23) is needed. Note that  $\gamma$  depends on  $u_{pp}$  and the function of  $\phi(r)$  depends on  $\gamma$ . The quantities  $m_1$  and  $m_2$ , which are required to obtain  $u_{pp}$  are also functions of  $\gamma$ . Since we do not know the value of  $\gamma$  a priori, an iterative procedure is required to obtain its correct value. The iterative procedure of Vallabhan and Das (1988) is employed here. The procedure is composed of the following steps.

- (1) Assuming  $\gamma = 1.0$ ;
- (2) Calculate  $m_1$  and  $m_2$  from eqn (8);
- (3) Calculate the displacement magnitude  $u_{pp}$  along the pile by solving eqn (19), with the boundary conditions eqns (20)–(21). Finite difference method (FDM) (Desai and Christian, 1977) is used here;
- (4) Use the results of (3) and calculate the new value of  $\gamma$  by using eqn (25);
- (5) Use the new value of  $\gamma$  and repeat steps 2–4. Iteration is continued until the difference between the  $i$ th and  $(i+1)$ th value of  $\gamma$

$$|\gamma_{i+1} - \gamma_i| \leq 0.001 \quad (25)$$

Finally, the displacement magnitude of the pile  $u_{pp}$  and the soil displacement in the  $r$ -direction,  $\phi(r)$ , can be obtained.

A computer program was written using FORTRAN 77 on IBM PC. Using the program, it is very easy to conduct the above iteration procedure.

## 5. Comparisons with field tests

To demonstrate the validity of the proposed model, the method is applied to calculate the performance of field test piles.

### 5.1. Case 1: Static load test piles

Matlock (1970) reported a field pile test at the area of Austin Lake. The pile length and diameter are respectively 12.8 and 32.4 cm. The pile stiffness is  $3.132 \times 10^4 \text{ kN m}^2$ . The average shear strength is 0.308 MPa. According to Poulos and Davis (1980), the secant Young's modulus  $E_g = (15-95)c_u$ , with an average value of  $40 c_u$  is used. The theoretical and experimental results of the pile displacement at the ground surface are shown in Fig. 2. It can be seen that the pile displacement can be well calculated if the soil modulus is appropriately chosen in the range  $E_s = (15-95)c_u$ .

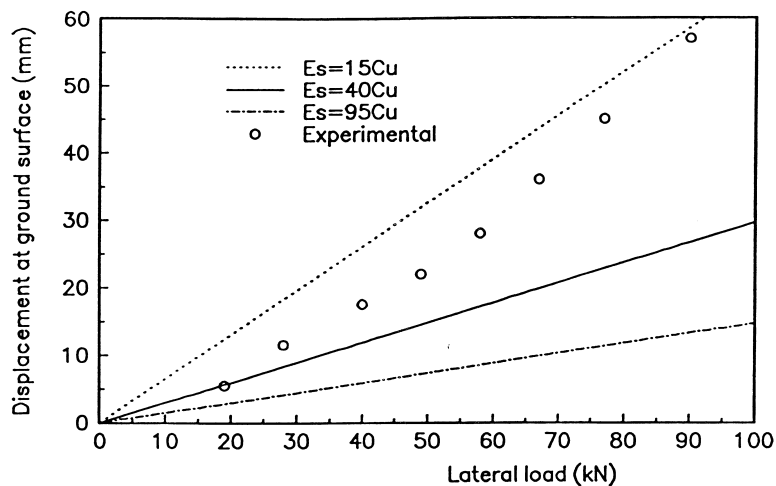


Fig. 2. Pile displacement at ground surface (Matlock, 1970).



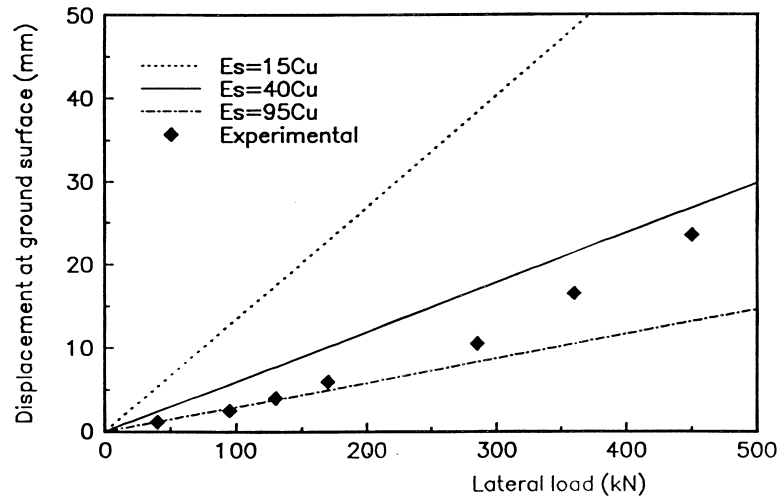


Fig. 3. Pile displacement at ground surface (Reese et al. 1974).

Reese et al. (1974) reported a field pile test at the area of Manor, Texas. The pile length and diameter are respectively 15.2 and 61.0 cm. The pile stiffness is  $4.110 \times 10^5$  kN m<sup>2</sup>. The average shear strength is 0.115 MPa. According to Poulos and Davis (1980), the soil modulus can be chosen as 1.725–10.925 MPa, with an average value of 4.6 MPa. The theoretical and experimental results of the pile displacement at the ground surface are shown in Fig. 3. It can also be seen that the pile displacement can be well calculated if the soil modulus is appropriately chosen in the range  $E_s = (15-95)c_u$ .

### 5.2. Case 2: dynamic load test piles

El-Marsafawi et al. (1992) reported horizontal vibration tests on a single pile. The tests were conducted at the pile research site at the Institute of Engineering Mechanics, Harbin, China. The cast-in-place reinforced concrete pile had a diameter of 0.32 and 7.5 m in length, connected by a 0.3-m-deep rigid reinforced concrete cap. The cap had a mass of 740 kg with its bottom surface situated 0.1 m above the ground surface. Figure 4a shows the layout of the pile and the cap. The pile properties were evaluated as: Young's modulus =  $1.96 \times 10^{10}$  N/m<sup>2</sup>, and the specific weight =  $2.45 \times 10^4$  N/m<sup>3</sup>. The soil at the site was a relatively homogenous sandy clay with yellow and brown coloring. The measured in situ shear wave velocity and mass density profiles are shown in Fig. 4b. The water table was 20 m below the ground surface. Poisson's ratio was taken as 0.3.

An exciter with two counter-rotating eccentric masses was fixed in the cap and used to produce horizontal harmonic excitation. The mass of the exciter was  $m_c = 120$  kg. The center of the cap-exciter system was 0.1 m below the cap surface. The exciting force acted 0.2 m above the cap surface in the *Y*-direction (Fig. 4a). The excitation forces is given by

$$q_0(t) = (m_c e) \Omega^2 \cos \Omega t \quad (26)$$

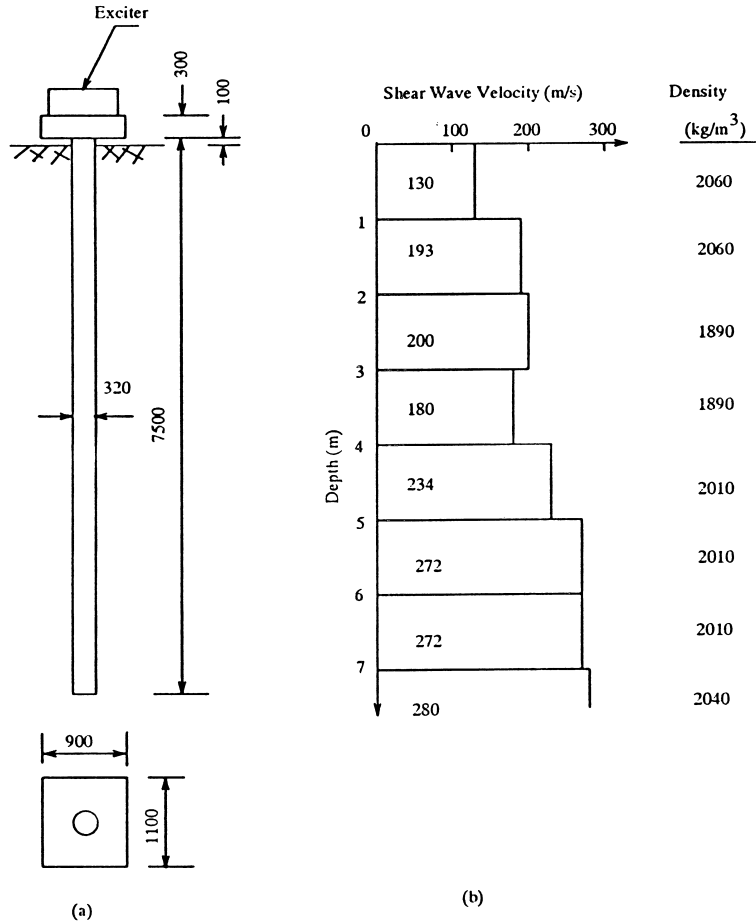


Fig. 4. (a) Layout of single pile test; (b) soil profile at the test site.

The experimental horizontal responses are shown as discrete points in Fig. 5. The normalized response amplitude is defined as

$$A_m = \left( \frac{m}{m_c e} \right) u_{pp} \tag{27}$$

where  $m$  = the total mass of the cap-exciter system.

The comparisons of the theoretical and experimental results of the pile displacement at the ground surface are shown in Fig. 5. A better prediction can be obtained when a reduced soil modulus is used.

**6. Parametric analysis**

The parameters investigated in this study are  $L/R$ ,  $E_p/E_s$ ,  $\rho_s/\rho_p$  and  $v$ . For the present study, various parameters within a range of practical values are used to illustrate their influences on the pile behavior.

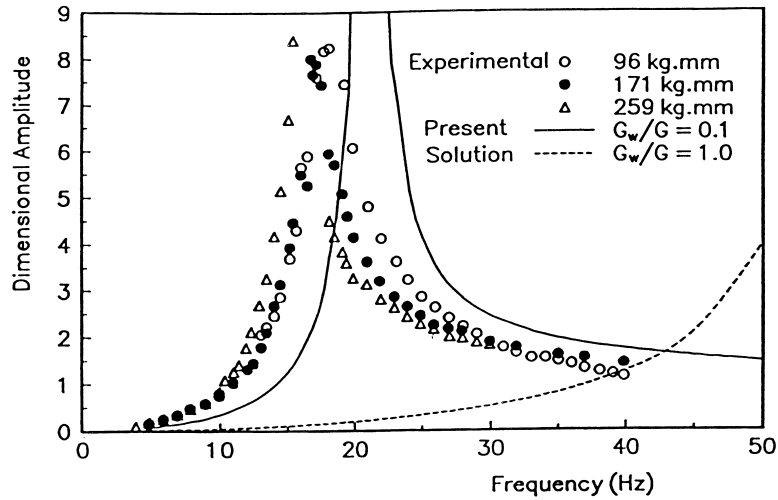


Fig. 5. Horizontal response of concrete single pile (El-Marsafawi, 1992).

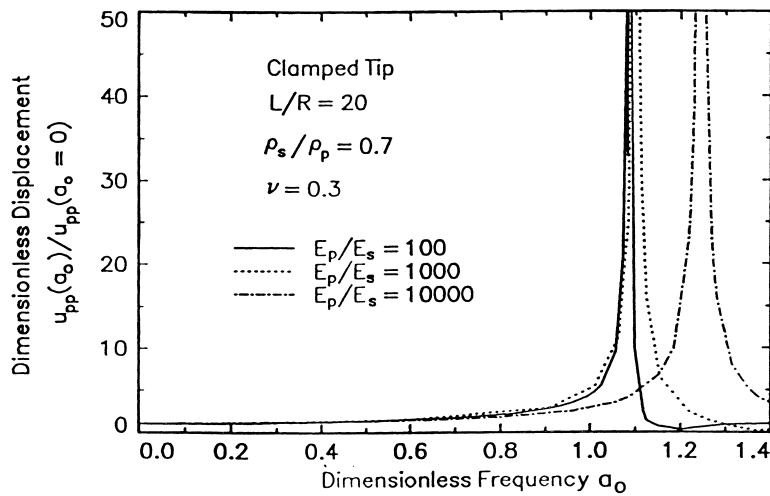


Fig. 6. Nondimensional displacement of the pile at different frequency.

### 6.1. Frequency response of pile head

The frequency response of pile head to horizontal excitation for a pile with clamped tip at different values of  $E_p/E_s$  are shown in Fig. 6. As expected, with the increase of  $E_p/E_s$ , the pile displacement increases. It can also be seen that the resonant dimensionless frequency  $a_0$  increases as  $E_p/E_s$  increases.

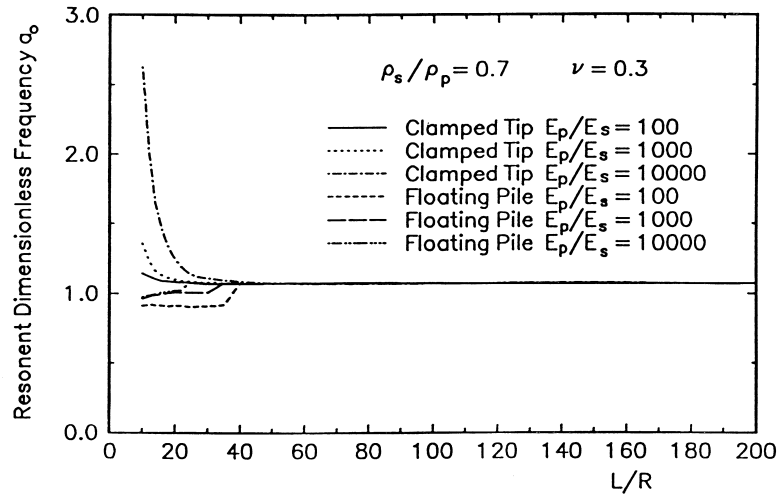


Fig. 7. Variation of resonant dimensionless frequency with  $L/R$  at different  $E_p/E_s$ .

## 6.2. Effects of various parameters on the pile behavior

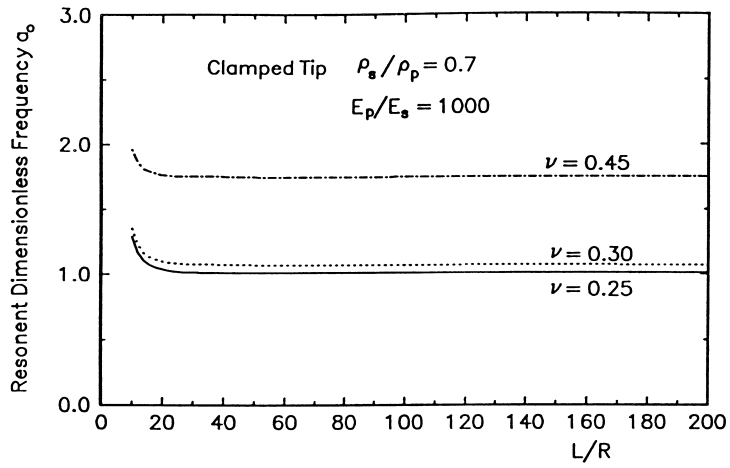
The effects of slenderness ratio  $L/R$  on the resonant dimensionless frequency  $a_0$  at different  $E_p/E_s$  are shown in Fig. 7. When  $L/R$  is greater than 40, the resonant dimensionless frequency  $a_0$  is the same whether the pile tip is clamped or floating. It means that pile tip conditions have no effect on the pile behavior when  $L/R$  is greater than 40. When  $L/R$  is smaller than 40,  $L/R$  has a great influence on the resonant dimensionless frequency  $a_0$ . For a pile with clamped tip, the resonant dimensionless frequency  $a_0$  decreases as  $L/R$  increases, the effect of  $L/R$  becoming more significant as  $E_p/E_s$  increases. The resonant dimensionless frequency  $a_0$  also increases as  $E_p/E_s$  increases. For a pile with floating tip, the resonant dimensionless frequency  $a_0$  increases as  $L/R$  increases, the effect of  $L/R$  becoming more significant as  $E_p/E_s$  decreases. The resonant dimensionless frequency  $a_0$  also increases as  $E_p/E_s$  decreases.

The effects of soil Poisson's ratio  $\nu$  on the resonant dimensionless frequency  $a_0$  with  $\rho_s/\rho_p = 0.7$  and  $E_p/E_s = 1000$  are shown in Fig. 8. The resonant dimensionless frequency  $a_0$  increases as soil Poisson's ratio  $\nu$  increases whether the pile tip is clamped or floating.

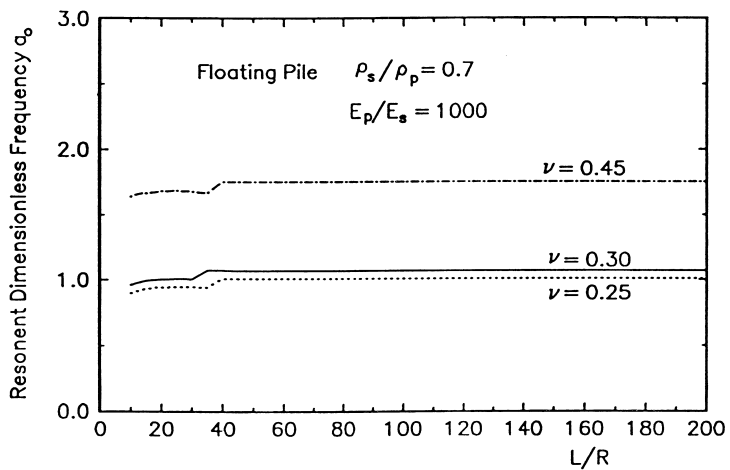
The effect of  $\rho_s/\rho_p$  on the resonant dimensionless frequency  $a_0$  with  $\nu = 0.3$  and  $E_p/E_s = 1000$  are shown in Fig. 9. The resonant dimensionless frequency  $a_0$  increases as  $\rho_s/\rho_p$  increases whether the pile tip is clamped or floating.

## 7. Summary and conclusions

An analytical approach for the determination of the response of a single cylindrical pile subjected to lateral dynamic load is presented. The method uses variational principle to obtain the governing differential equations of the soil and the pile system. A non-dimensional parameter  $\gamma$  associated with the characteristics of the pile, the soil and the loading is used to represent the elastic foundation.



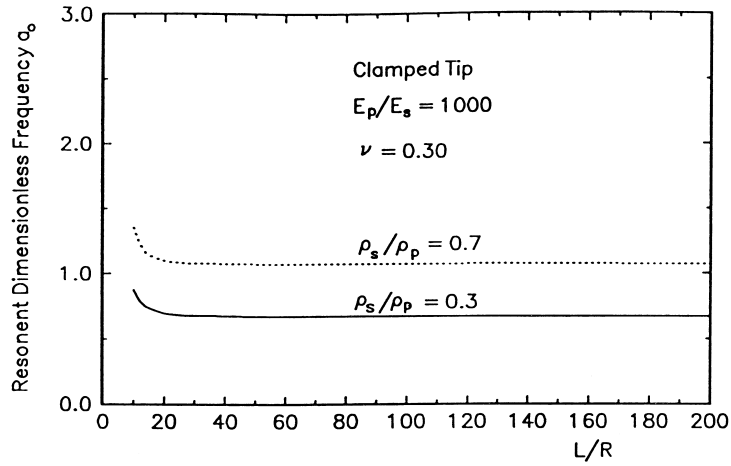
(a)



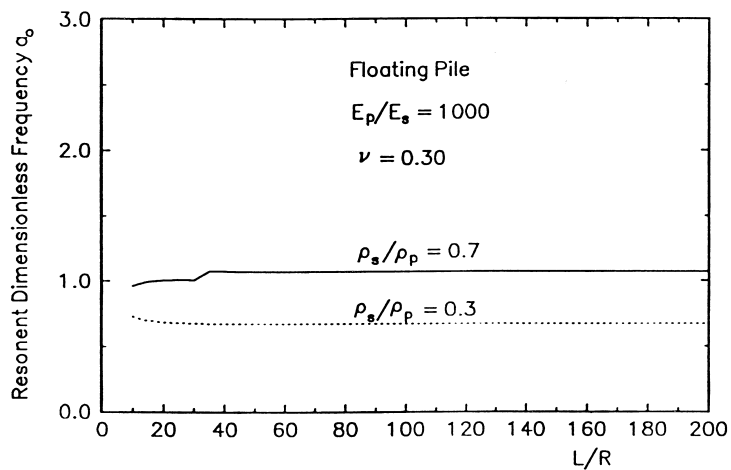
(b)

Fig. 8. Effect of soil Poisson's ratio on resonant dimensionless frequency.

The parameter  $\gamma$  can be determined by using an iterative procedure. The classical finite difference method is used to solve the differential equations. Analyses of two static field test piles show that the pile displacement can be well calculated if the soil modulus is appropriately chosen in the range  $E_s = (15-95) c_u$ . Analyses of one dynamic field test pile shows that a better prediction can be obtained when a reduced soil modulus is used. However, the discrepancy noticed in the comparison



(a)



(b)

Fig. 9. Effect of pile density on resonant dimensionless frequency.

between the analytical and experimental results is mainly due to the assumptions made in developing the analytical model.

The parametric studies with regard to frequency response of the pile head and the resonant frequency of the pile-soil system show that:

- (1) When  $L/R$  is greater than 40, the resonant dimensionless frequency  $a_0$  is the same whether the pile tip is clamped or floating.

- (2) When  $L/R$  is smaller than 40,  $L/R$  has a great influence on the resonant dimensionless frequency  $a_o$ . For a pile with clamped tip, the resonant dimensionless frequency  $a_o$  decrease as  $L/R$  increases, the effect of  $L/R$  becoming more significant as  $E_p/E_s$  increases. The resonant dimensionless frequency  $a_o$  also increases as  $E_p/E_s$  increases. For a pile floating tip, the resonant dimensionless frequency  $a_o$  increases as  $L/R$  increases, the effect of  $L/R$  becoming more significant as  $E_p/E_s$  decreases. The resonant dimensionless frequency  $a_o$  also increases as  $E_p/E_s$  decreases.
- (3) The resonant dimensionless frequency  $a_o$  increases as soil Poisson's ratio  $\nu$  increases whether the pile tip is clamped or floating.
- (4) The resonant dimensionless frequency  $a_o$  increases as  $\rho_s/\rho_p$  increases whether the pile tip is clamped or floating.

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